



# **Seconda parte (5-9-2011)**

## Definizione dei termini ‘ $K$ ’

Sia  $\psi$  una quantità trasportata dal moto:

$$\partial_t \psi = K_\psi = -\frac{1}{\rho} \partial_z (\Phi)$$

$$\Phi = -\rho K \partial_z \psi$$

$$\Phi_S = -\rho_S C_D (\psi_{NLEV} - \psi_S)$$

$\Phi$  Flusso turbolento (positivo se ‘upward’)

$\Phi_S$  flusso turbolento al suolo

Chiusura E-l:

$$K = l_m \sqrt{C_E \bar{E}}$$

$l_m$  mixing length: stable  $\rightarrow$  Blackadar modified  
unstable  $\rightarrow$  Bougeault 1986 (modified)

$$u = \bar{u} + u', \quad v = \bar{v} + v', \quad w = \bar{w} + w', \quad \psi = \bar{\psi} + \psi', \quad P', \rho' \approx 0$$

$$\bar{E} = \frac{1}{2} (\overline{u'u'} + \overline{v'v'} + \overline{w'w'})$$

## Equazione per l'energia cinetica turbolenta

$$\frac{d\bar{E}}{dt} = K_E - \overline{u_i' u_j'} \frac{\partial \bar{u}_i}{\partial x_j} + \frac{g}{\theta_V} \overline{w' \theta'_V} - \varepsilon$$

Ipotesi sul termine dissipativo:

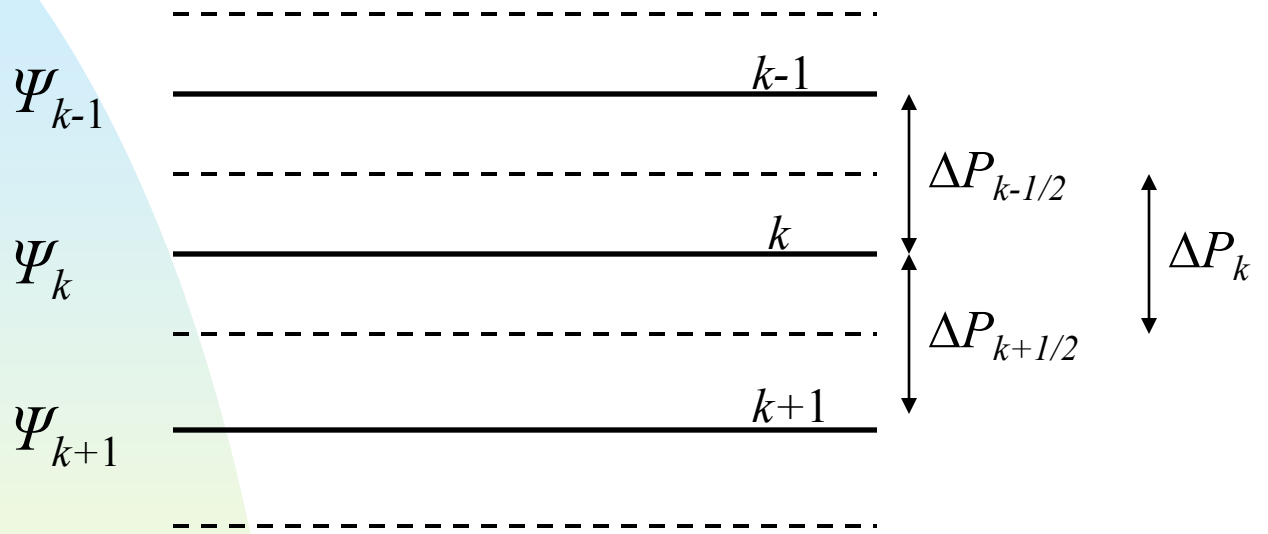
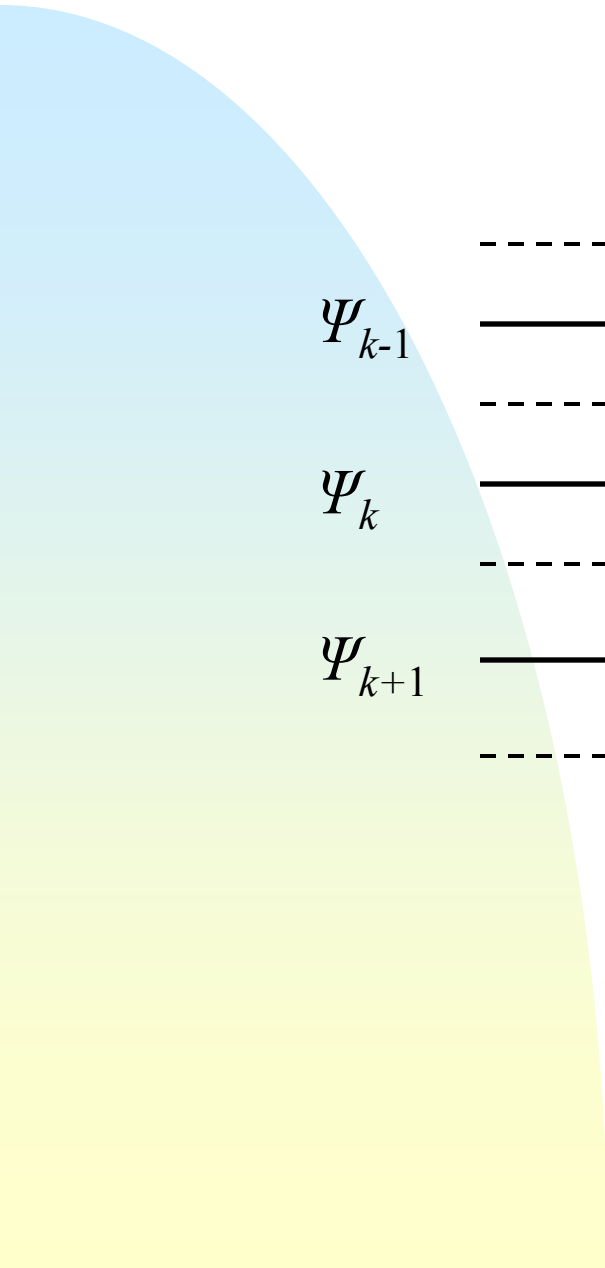
$$\varepsilon = \frac{1}{l_m} \left( \sqrt{C_E \bar{E}} \right)^3$$

## Closure hypothesis: Turbulent fluxes of a scalar quantity

$$\begin{array}{l} \overline{u'\psi'} \\ \overline{v'\psi'} \\ \overline{w'\psi'} \end{array} = -K \vec{\nabla} \overline{\psi}$$

## Closure hypothesis: Turbulent fluxes of a vector field

$$\overline{u_i' u_j'} = \frac{2}{3} \delta_{ij} \overline{E} - K \left( \frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right)$$

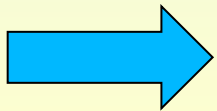


## Schema 'backward' implicito

$$\psi_k^{n+1} = \psi_k^n - c_k (\psi_{k+1}^{n+1} - \psi_k^{n+1}) + a_k (\psi_k^{n+1} - \psi_{k-1}^{n+1})$$

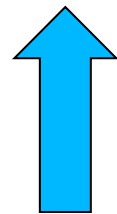
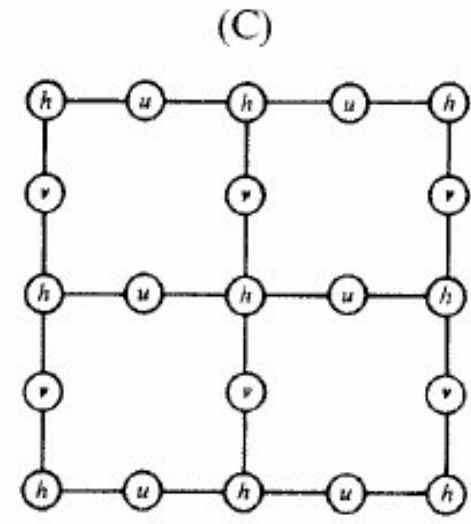
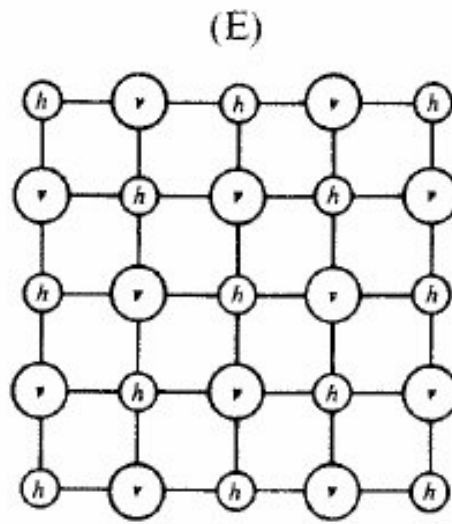
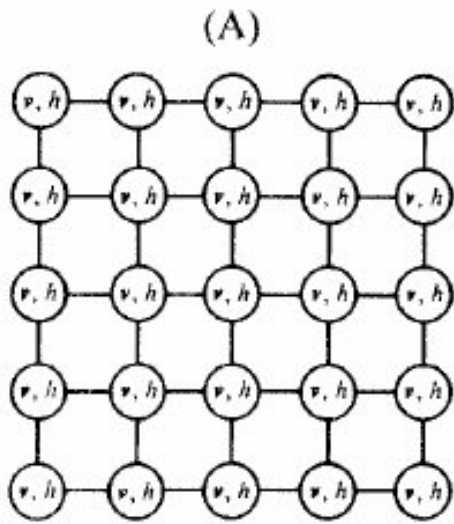
$$c_k = -\frac{\Delta t g^2 \Delta P_{k+\frac{1}{2}} K_{k+\frac{1}{2}}}{\Delta P_k \Delta \Phi_{k+\frac{1}{2}}^2}, \quad k = N_{TOP}, \dots, N_{LEV} - 1$$

$$a_k = -\frac{\Delta t g^2 \Delta P_{k-\frac{1}{2}} K_{k-\frac{1}{2}}}{\Delta P_k \Delta \Phi_{k-\frac{1}{2}}^2}$$



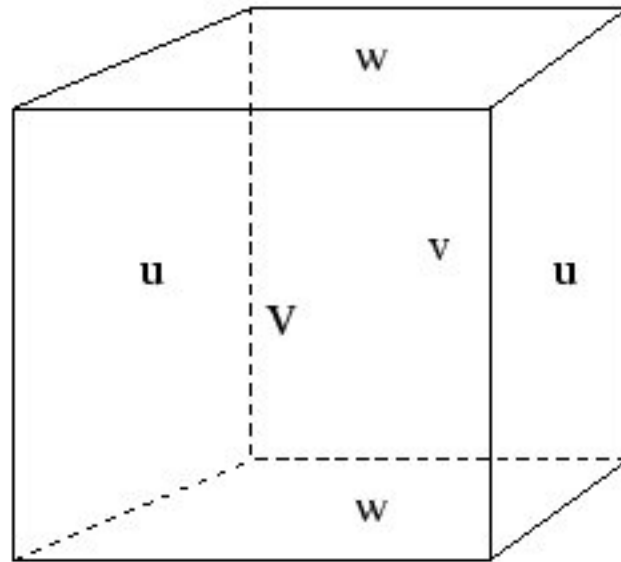
$$a_k \psi_{k-1}^{n+1} + (1 - a_k - c_k) \psi_k^{n+1} + c_k \psi_{k+1}^{n+1} = \psi_k^n$$

# Discretizzazione orizzontale





## Discretizzazione verticale ('Lorenz')



Three dimensional staggering of velocity components. This facilitates the natural discretization of the continuity and tracer equations.

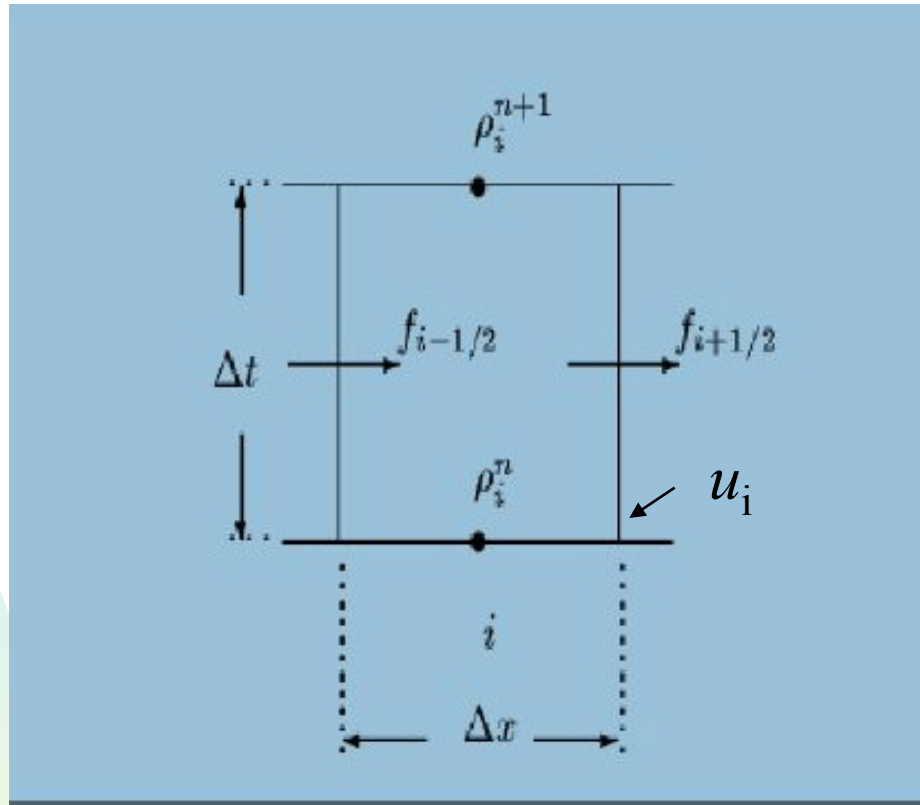
# Schema di avvezione 'WAF'

Toro, E.F. *A Weighted Average Flux method for hyperbolic conservation laws*. Proc. Roy. Soc. Lond. A 423, pp 401-418, 1989.

Toro, E.F. *The Weighted Average Flux method applied to the Euler equation*. Phil. Trans. R. Soc. Lond. A 341, pp 499-500, 1992.

Godunov, S.K., *A difference scheme for numerical computation of discontinuous solution of hydrodynamic equation*, Math. Sbornik, 47, pp 271-306, in Russian, Translated US Joint Publ. Res. Serv. JPRS 7226 (1969), 1959.

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0$$



$$f = \rho u$$

$$\rho_i^{n+1} = \rho_i^n - \frac{\Delta t}{\Delta x} (f_{i+1/2} - f_{i-1/2}) = 0$$

‘WAF limiter’ :

$$f_{i+1/2} = \frac{1}{2}(1 + \phi)u_i\rho_i^n + \frac{1}{2}(1 - \phi)u_i\rho_{i+1}^n$$

$$\phi = \phi(c, r)$$

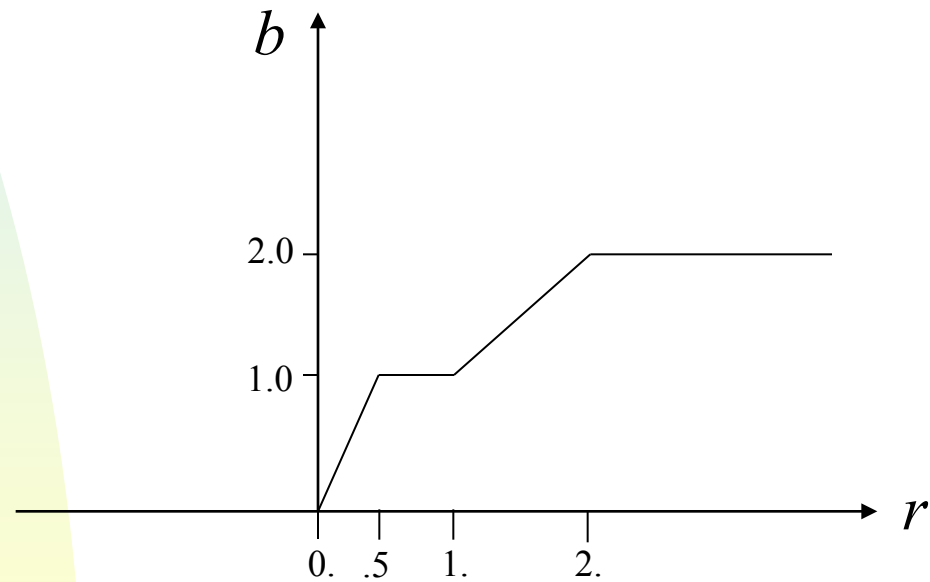
$$c = u_i \frac{\Delta t}{\Delta x}$$

$$r = \frac{(\rho_i^n - \rho_{i-1}^n)}{(\rho_{i+1}^n - \rho_i^n)}, \quad c \geq 0$$

$$r = \frac{(\rho_{i+2}^n - \rho_{i+1}^n)}{(\rho_{i+1}^n - \rho_i^n)}, \quad c \leq 0$$

$$\phi_{i+1/2} = \text{sgn}(c) [1 + (|c| - 1)b(r)]$$

$$b(r) = \max(0, \min(2r, 1), \min(r, 2))$$



## 3D WAF in coordinate sferiche:

Hubbard and Nikiforakis, 2003:

‘A three-dimensional adaptive, Godunov type model for global atmospheric flows I: tracer advection on fixed grids’.

*Monthly Weather Review*, **131**, 1848-1864.

$$\frac{\partial \rho}{\partial t} + \vec{V} \cdot \vec{\nabla} \rho = 0$$

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla}(\rho \vec{V}) + \rho \vec{\nabla} \cdot \vec{V}$$

Schema temporale 'time splitted' che conserva la massa

$$\rho^* = \rho^n - \Delta t \frac{\partial(\rho^n \dot{\sigma})}{\partial \sigma} + \rho^n \frac{\partial \dot{\sigma}}{\partial \sigma}$$

$$\rho^{**} = \rho^* - \Delta t \frac{\partial(\rho^* v)}{\partial y} + \rho^n \frac{\partial v}{\partial y}$$

$$\rho^{n+1} = \rho^{**} - \Delta t \frac{\partial(\rho^{**} u)}{\partial x} + \rho^n \frac{\partial u}{\partial x}$$

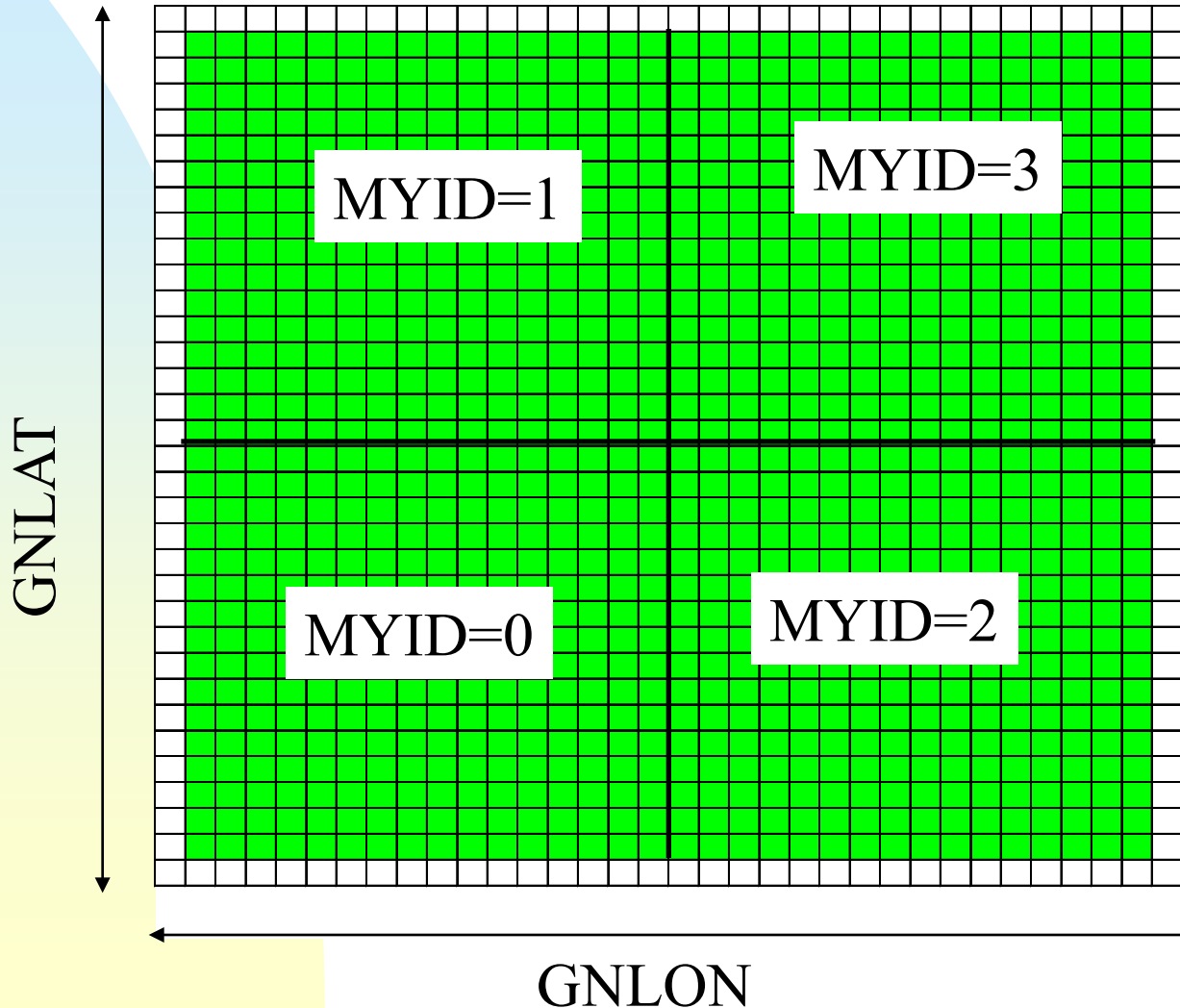
# Schema temporale generale di Bolam

- Time split
- ‘forward backward’ per la parte di onde di gravità (timestep  $DTSTEP/4$ )
- Avvezioni (timestep  $DTSTEP$ )
- Fisica (timestep  $DTSTEP$ )
- Rilassamento alle condizioni al contorno

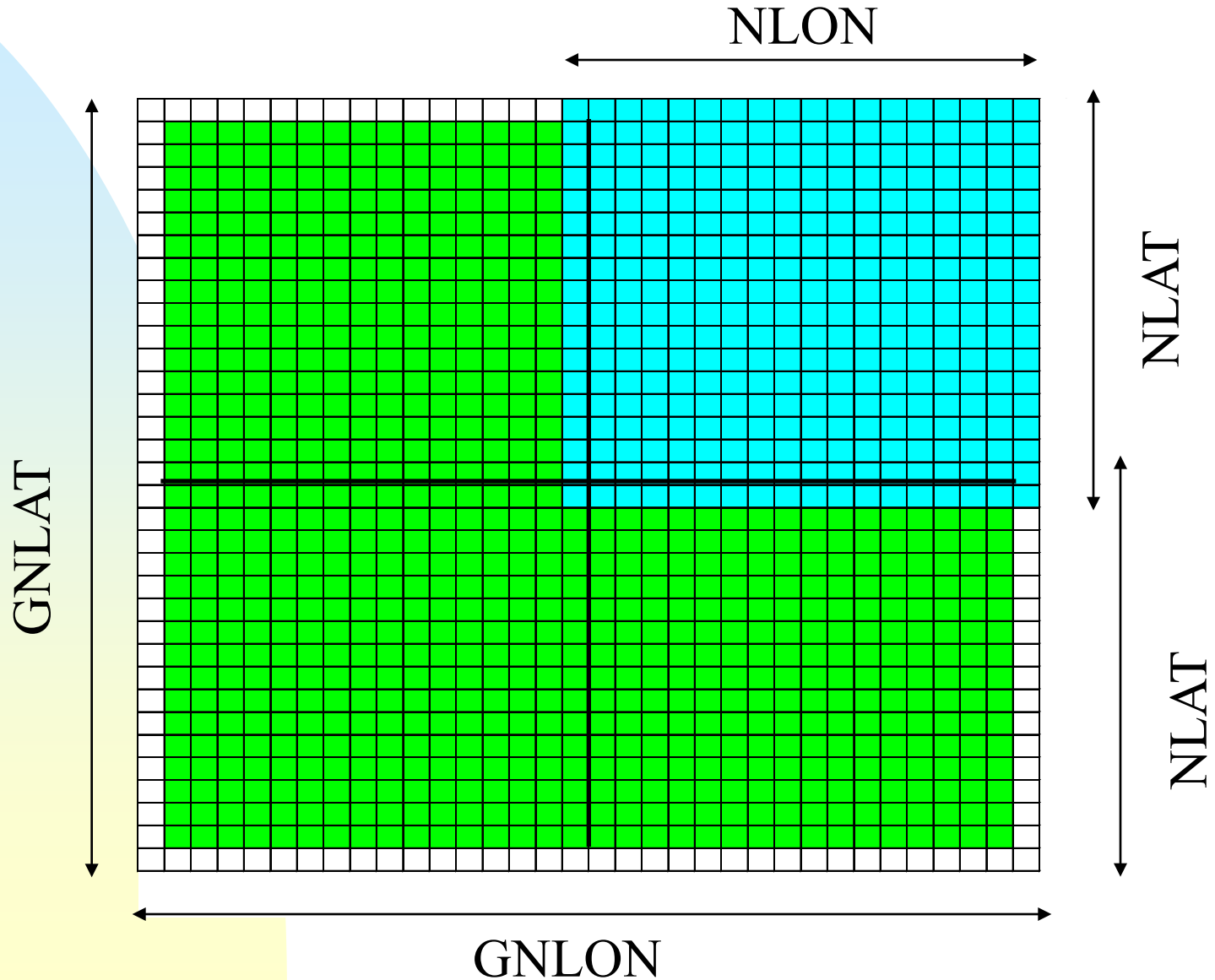


# Parallelismo di Bolam

'Domain decomposition'



Per facilitare il calcolo delle derivate si introduce una 'ghostline' per ogni sottodominio



NPROCSX, NPROCSY

$$NLON=(GNLON-2)/NPROCSX+2$$

$$NLAT=(GNLAT-2)/NPROCSY+2$$

$$GNLON=(NLON-2)*NPROCSX+2$$

$$GNLAT=(NLAT-2)*NPROCSY+2$$

(NLON e NLAT numeri pari)

# 'Speedup' della versione parallela

- Dipende dall'architettura della macchina
- Dipende dalla possibilità di utilizzare hyper-threading (two virtual processes for each processor core)
- Dipende da come si 'spalma' il dominio fisico sui processori
- Mediamente con 16 processi uno speedup di 8 volte
- Tende a saturare per numero elevato di processi ( $>100$ )