Numerical methods for wave/flow computations

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APAT

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Contents

1. Introduction to numerical models for large scale coastal problems

Numerical models for large scale coastal problems must be capable of dealing with wave generation, wave propagation and wave breaking. Generally the area interested by the simulation ranges from 100-200 km to 5-10 km in width and from 1 to 50 km in length (from the coast towards deep water) and also the required resolution could be very coarse or very fine. The main forces driving the wave propagation in this kind of problems are the wind and the interaction with the sea bottom, however also the boundary conditions that describe the waves entering the domain play an important role.

2. SWAN (Simulating WAves Nearshore)

SWAN is a third-generation wave model for obtaining realistic estimates of wave parameters in coastal areas, lakes and estuaries from given wind, bottom and current conditions, although it can be used on any scale relevant for wind-generated surface gravity waves. SWAN was designed specifically to simulate **coastal** wind waves, unlike other third – generation models such as WAM (WAMDI Group, 1988) and WAVEWATCH (Tolman, 1991) which were developed specifically for simulating waves in deep water. The basic scientific philosophy of SWAN is identical to that of WAM (Cycle 3 and 4) and it uses the same formulations for the source terms. However SWAN contains some additional formulations primarily for shallow water and the numerical techniques are very different.

2. SWAN (Simulating WAves Nearshore)

The following wave propagation processes are represented in SWAN:

- propagation through geographic space,
- refraction due to spatial variations in bottom and current,
- diffraction*,
- shoaling due to spatial variations in bottom and current,
- blocking and reflections by opposing currents,
- transmission through, blockage by or reflection against obstacles.

*Diffraction is modelled in a restricted sense, so the model should be used in areas where variations in wave height are large within a horizontal scale of a few wave lengths. However, the computation of diffraction in arbitrary geophysical conditions is rather complicated and requires considerable computing effort.

2. SWAN (Simulating WAves Nearshore)

The following wave generation and dissipation processes are represented in SWAN:

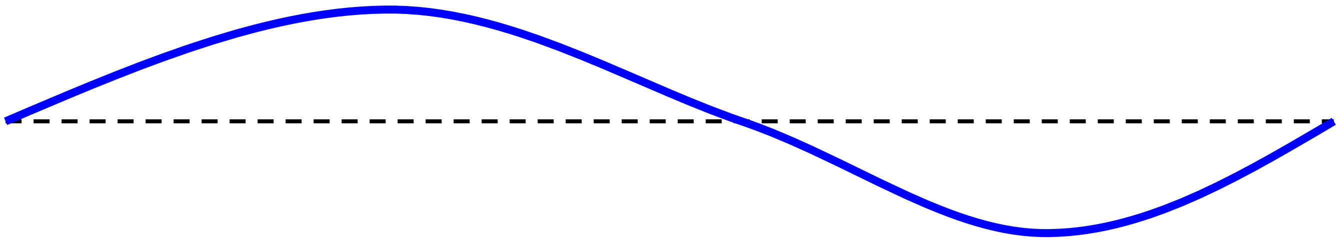
- generation by wind,
- dissipation by whitecapping,
- dissipation by depth-induced wave breaking,
- dissipation by bottom friction
- wave-wave interactions in both deep and shallow water.

3. SWAN governing equation

The main goal of the SWAN model is to solve the spectral action balance equation without any a priori restrictions on the spectrum for the evolution of wave growth. This equation represents the effects of spatial propagation, refraction, shoaling, generation, dissipation and nonlinear wave-wave interactions.

3. SWAN governing equation

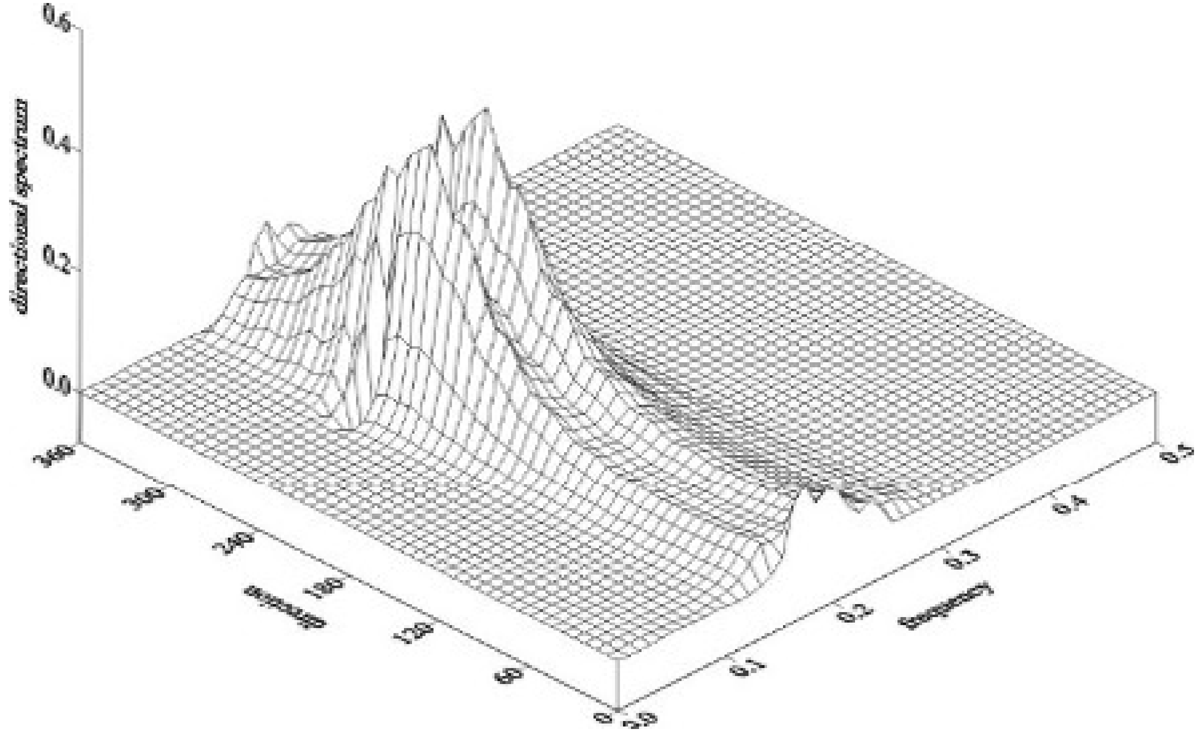
The energy associated with a monochromatic wave with frequency s_1 and direction q_1 is:



$$E(\sigma_1, \vartheta_1) = \frac{1}{8}\rho g H^2$$

3. SWAN governing equation

The energy associated with a superposition of monochromatic waves with different frequencies and directions is:



3. SWAN governing equation

The spectral action balance equation:

$$E(\sigma,\vartheta)$$

Energy

$$N(\sigma,\vartheta)=\frac{E(\sigma,\vartheta)}{\sigma}$$

Action density

$$\boxed{\underbrace{\frac{\partial N}{\partial t}}_{\mathrm{I}}+\underbrace{\frac{\partial(c_x N)}{\partial x}+\frac{\partial(c_y N)}{\partial y}}_{\mathrm{II}}+\underbrace{\frac{\partial(c_\sigma N)}{\partial \sigma}}_{\mathrm{III}}+\underbrace{\frac{\partial(c_\vartheta N)}{\partial \vartheta}}_{\mathrm{IV}}=\underbrace{\frac{S}{\sigma}}_{\mathrm{V}}}$$

I — Temporal variation

II — Spatial variation

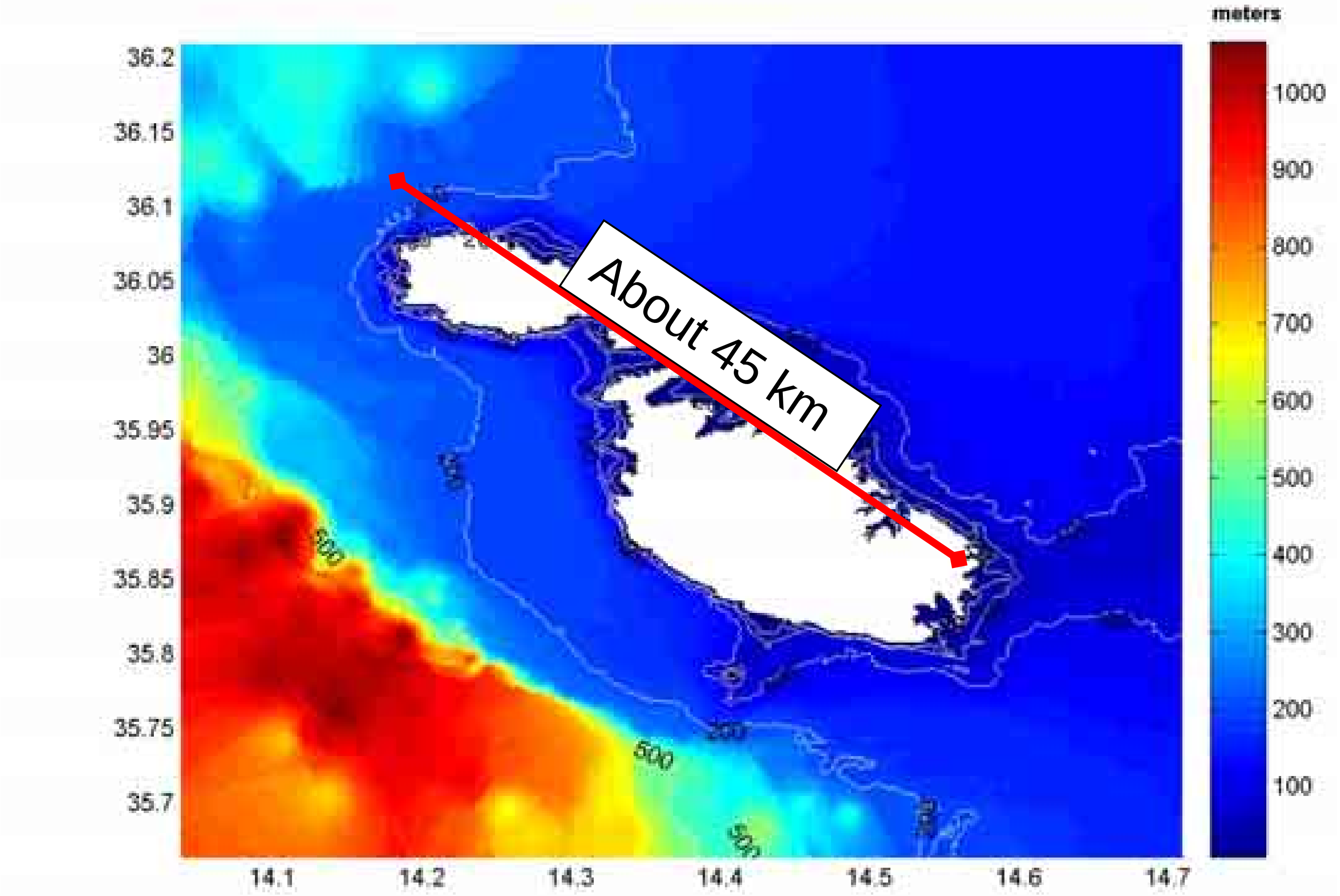
III — Variation in the frequencies' domain

IV — Variation in the directions' domain

V — Source term

4. SWAN implementation

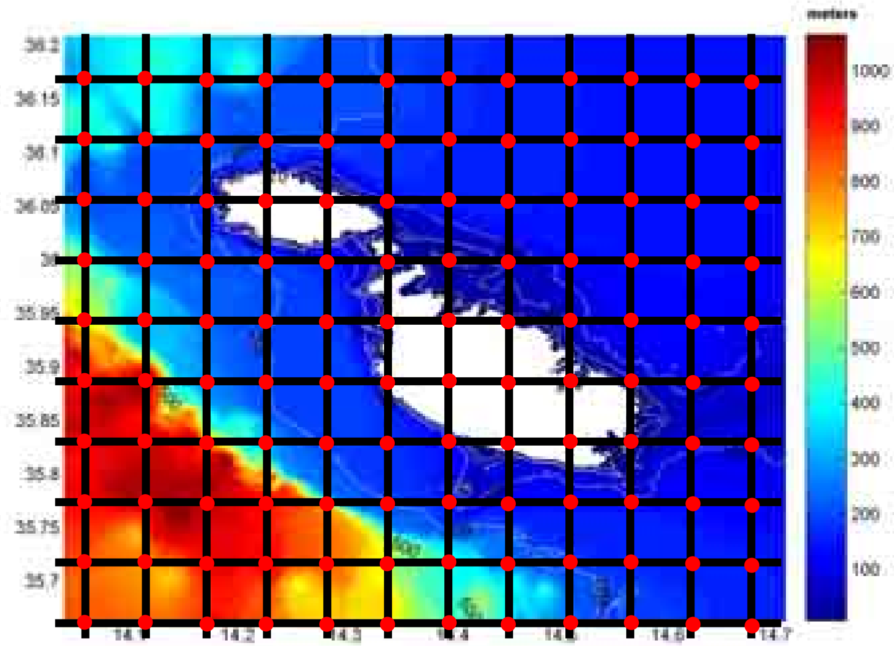
Bathymetry



4. SWAN implementation

Computational grid

The whole area is transformed into a computational grid. There will be inputs and outputs only at the nodes of the grid. The finer the grid the longer the computation.

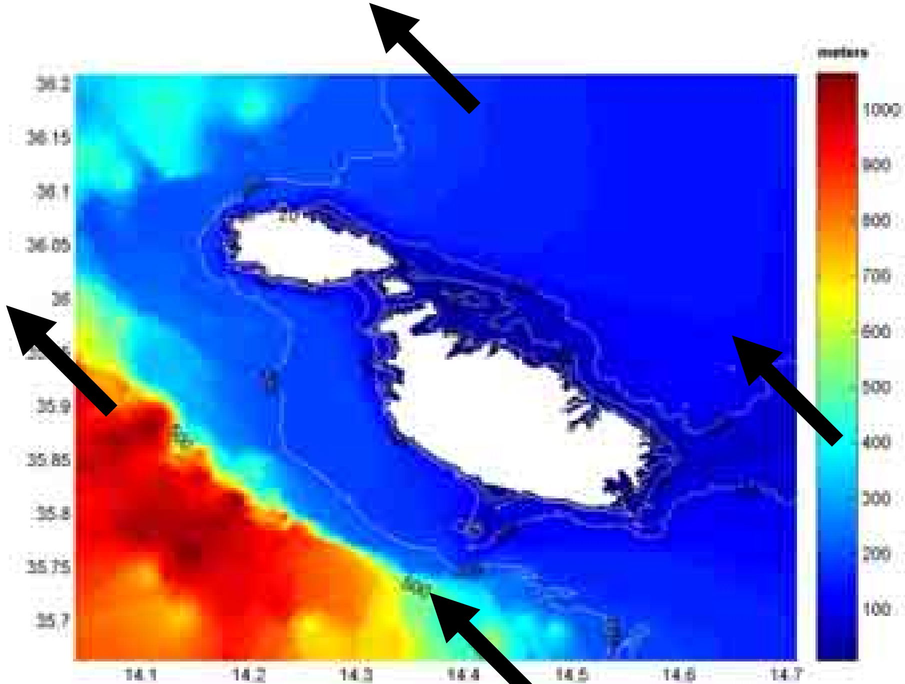


Typical cell size 50->200 m

4. SWAN implementation

Boundary conditions

In this case the sea conditions incoming at the border of the chosen domain come from the forecast of the WAM model.

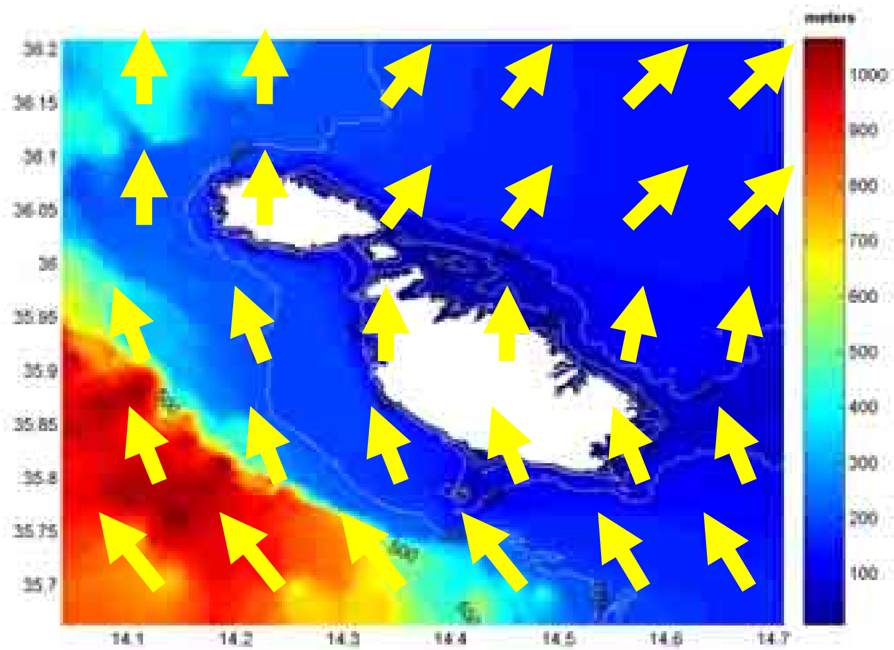


The SWAN model is not properly designed for the simulation of a whole basin, however when dealing with “closed” (or almost closed) domains the boundary conditions are not so important as a well reproduced wind field.

4. SWAN implementation

Wind

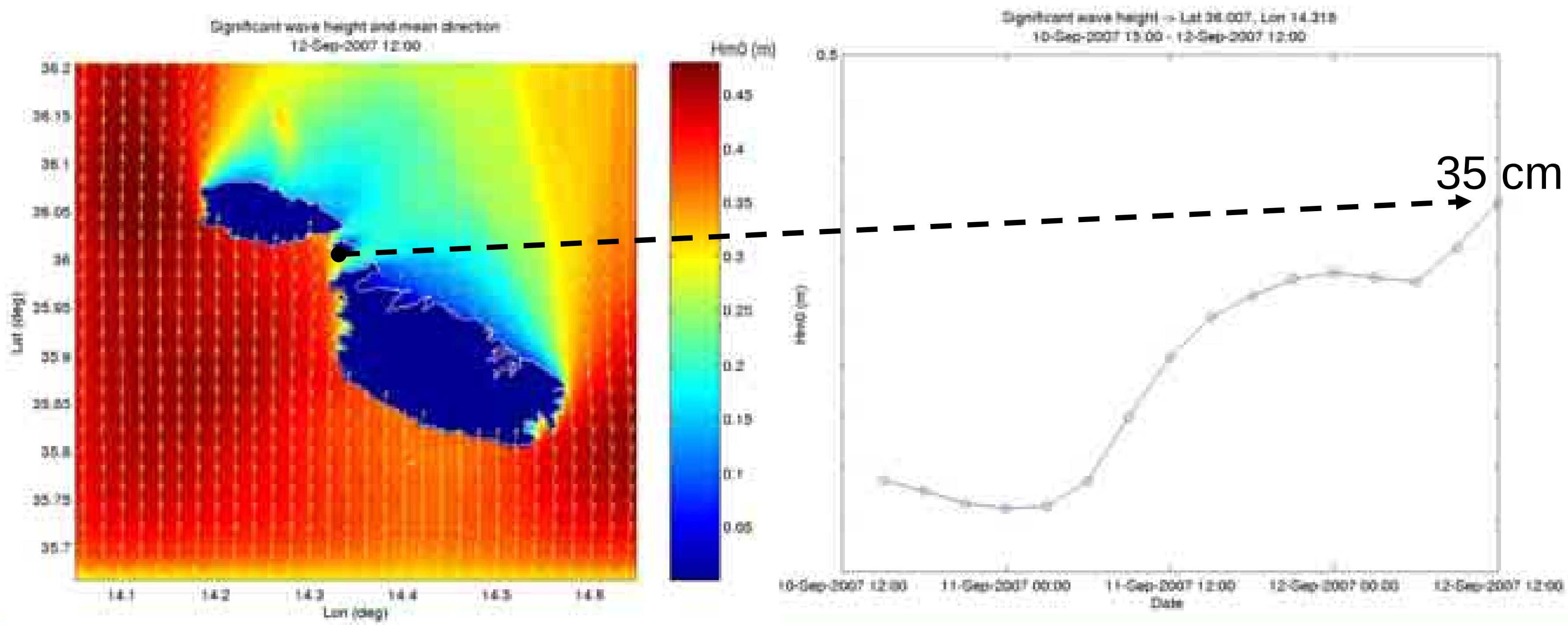
The wind field inside the domain comes from a meteorological model. However it is not mandatory to use a wind field. When dealing with small domains wind effects could be neglected because of a very small fetch length.



4. SWAN implementation

Model Run – Non stationary

A typical application for the SWAN model is the simulation of an **event**. This means that one tries to reproduce the effect of a particular **sequence** of sea conditions in order to evaluate how this event propagates toward the coast. This is what is done when using the SWAN as a forecasting model.



4. SWAN implementation

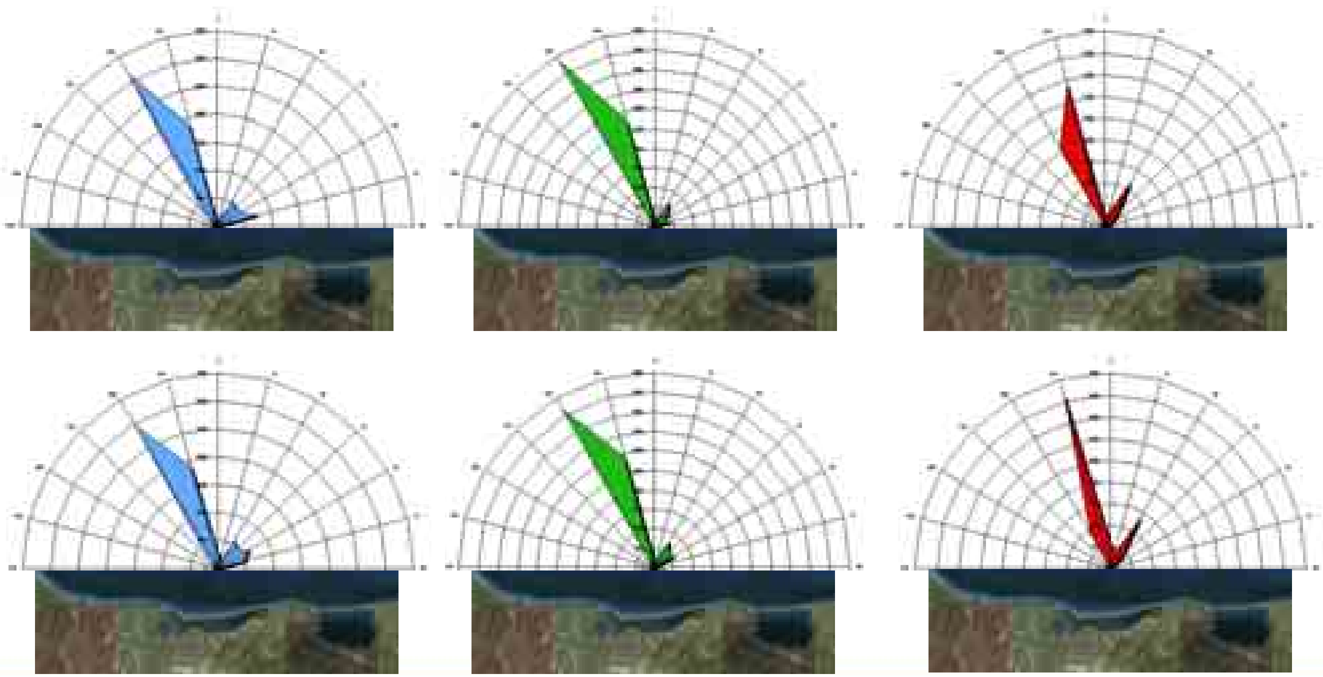
Model Run – Stationary

A stationary run is used to study the effects of a particular sea state or for climatological purposes. In this case **one (and only one) sea state** is propagated towards coast with fixed boundary conditions and wind field till the model obtains almost stable result in each cell of the computational grid.

Waves < 1 m | Waves 1-2 m | Waves > 2 m

Deep Water

Coast (depth 10 m)



4. SWAN implementation

Model Run – Nesting

The idea of nesting is to first compute the waves on a coarser computational grid for a larger region and then on a finer grid for a smaller region. The computation on the nested grid uses boundary conditions that are generated by the computation on the coarse grid. This two steps approach is useful for reducing the computational time. Instead of running the numerical model at its maximum resolution for the whole region even if we are interested only in a small included area, we make a coarse resolution run and then a high resolution one just for the area of interest, nested in the previous one. Thus the large scale is linked to the small scale.

5. Conclusions

When dealing with numerical models it is important to choose the right model for the phenomenon that must be simulated. Each models has some particular features that make it useful for representing some kind of phenomena. Large scale applications for coastal problems require models that can deal with domain ranging from one or two hundred kilometres to few kilometres in width and from 1 to 50 km in length. They must take into account effects from the interaction of the waves with the wind, with other waves or currents and with the sea bottom.

The SWAN, which has been briefly presented, is just one of these numerical models.

The bibliography

- The SWAN team (2007), SWAN User Manual, Delft University of Technology.
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