Water waves/jets. Linear wave theory, long waves, short waves, wave/coast interaction, jets, jet/wall interaction
Part II

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2. Linear wave theory: short waves
3. Formulation for long waves and currents
4. Jets

1. Introduction to small scale problems

1.1 General remarks

In the present framework the meaning of "small scale" is concerned with the scale of coastal engineering design (100m - 1km).

1. Introduction to small scale problems

The related hydrodynamic problems can be tackled by experimental techniques or by numerical methods. Sometimes, both the approaches are implemented, aiming at validating the numerical results. However, several input data are necessary:

•large scale analysis results: wave climate, general circulation;

•field data collection: remote sensing flow visualization, wind, water quality, river flows;

•bathymetry (current and previous ones), shoreline history, accurate geometry of marine structures

Fig.1 shows an example of river mouth flow visualization by infrared techniques: dark blue indicates cold river waters, red indicates hot marine waters.

1. Introduction to small scale problems

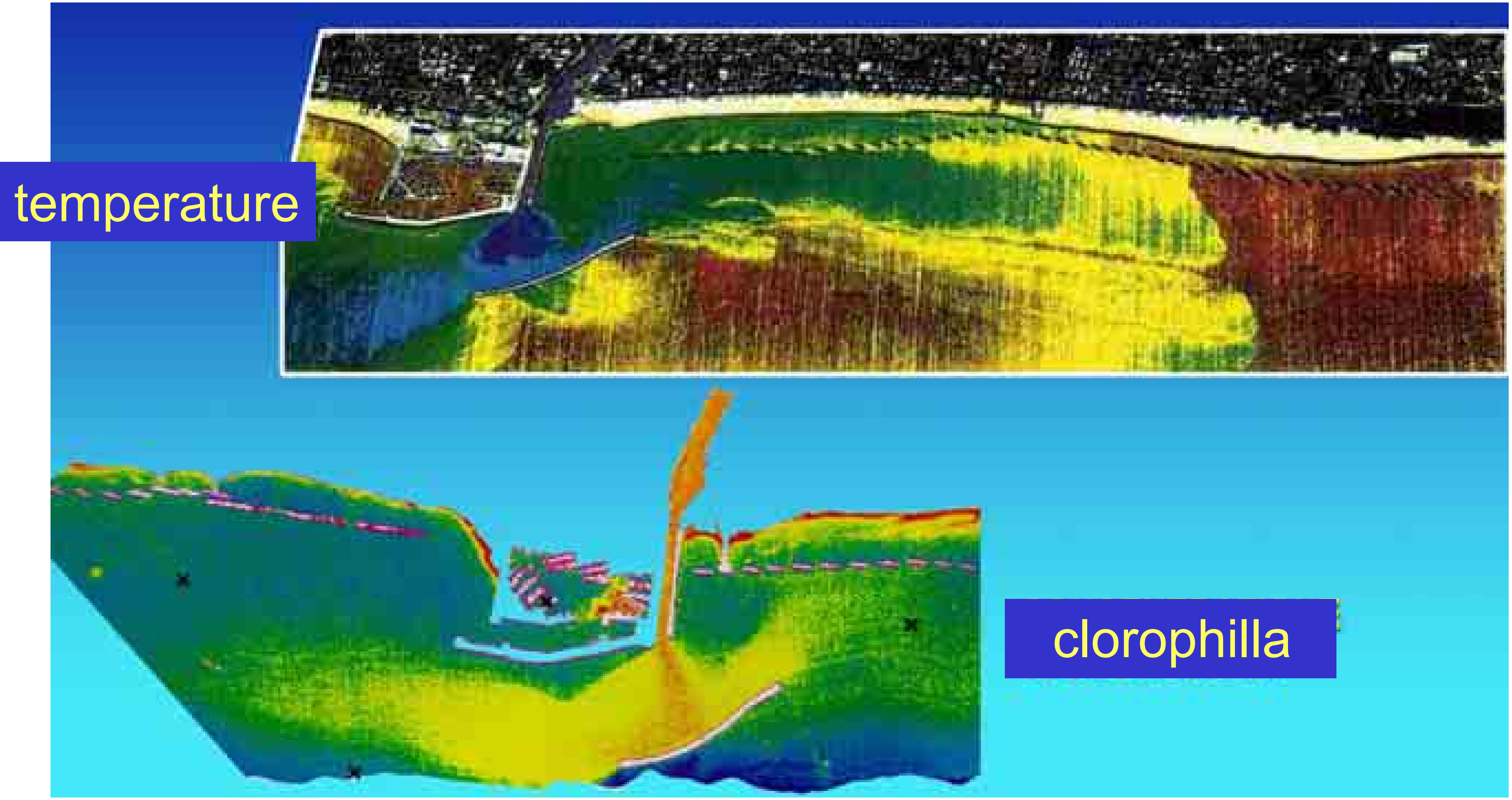


Figure 1: Pescara harbour: river flow visualization.

1. Introduction to small scale problems

The initial step of data collection is very important; though very advanced models can be implemented to compute fluid-structure interactions, if the data (wave climate, geometry) are not accurate, results can be very misleading.

It is worth noticing that coastal engineering problems are very delicate to tackle for some reasons:

•the coastal environment is, strictly speaking, the final recipient of waste waters; modifications of local flows can be very important for the diffusion of pollutants in the sea;

•coastal engineering structures give rise not only to some (desired) local effects, but can also produce unexpected (undesired) effects in the neighbourhood.

2. Linear wave theory: short waves

The exact formulation for incompressible free surface flow can be given by mass and momentum conservation equations (Navier Stokes):

$$\nabla \cdot \vec{u} = 0 \qquad\qquad \vec{u} = (u, v, w))$$

$$\rho\left(\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u}\right) = -\nabla p + \rho \vec{g} + \mu \nabla^2 \vec{u}$$

In which ρ, $u(x,y,z,t)$, $p(x,y,z,t)$, g, μ are respectively density, velocity, pressure, gravity acceleration and viscosity. These equations can be solved by implementing suitable initial and boundary conditions. The problem stated in these terms is very difficult to tackle, and this formulation is nowadays used only for very small scale test cases (of the order of 1-10 m). On the other hand, wave generation and propagation problems can be more suitably studied by using potential flow models, since wave motion (in the absence of breaking) is driven by conservative field forces.

2. Linear wave theory: short waves

The flow is irrotational, and the velocity field can be expressed as the gradient of a scalar function *Φ(x,y,z,t)*:

$$\vec{u} = \nabla \phi$$

Taking into account the conservation of mass law, Laplace equation is obtained, to be considered with linear free surface boundary condition:

$$\nabla^2 \phi = 0$$

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z}$$

$$\frac{\partial \phi}{\partial t} = -\left(g\eta + \frac{p_0}{\rho} \right)$$

with z=*η(x,y,t)* free surface elevation, $p_0(x,y,t)$ atmospheric pressure.

2. Linear wave theory: short waves

Finally, the impermeability condition at rigid boundaries (bottom, marine structures) reads:

$$\frac{\partial \phi}{\partial n} = 0$$

This mathematical formulation, effective for small amplitude waves, once implemented by means of a suitable numerical method, can give, in principle, the solution of any water wave problem (wave generation by atmospheric pressure gradients, wave propagation, refraction, reflection and diffraction).

It is worth noticing that all the wave features concerning fluid-structure interaction, as reflection, refraction and diffraction are summarized in the impermeability condition. On the other hand, this approach cannot deal with breaking effects, strongly concerned with viscosity and rotational flow features.

2. Linear wave theory. Short waves

In the case of uniform depth *h*, the simplest solution for the linear problem described above is the long crested wave (for example, propagating in the *x* direction):

$$\eta(x,t) = \frac{H}{2}\cos(kx - \omega t)$$

$$\phi(x,z,t) = \frac{gH}{2\omega}\frac{\cosh[k(z+h)]}{\cosh(kh)}\sin(kx-\omega t)$$

with *H*, *k*, *ω* wave amplitude, wave number and radian frequency. The related velocity components are (see fig.2):

$$u(x,z,t) = \frac{gkH}{2\omega}\frac{\cosh[k(z+h)]}{\cosh(kh)}\sin(kx-\omega t)$$

$$w(x,z,t) = \frac{gkH}{2\omega}\frac{\sinh[k(z+h)]}{\cosh(kh)}\sin(kx-\omega t)$$

2. Linear wave theory: short waves

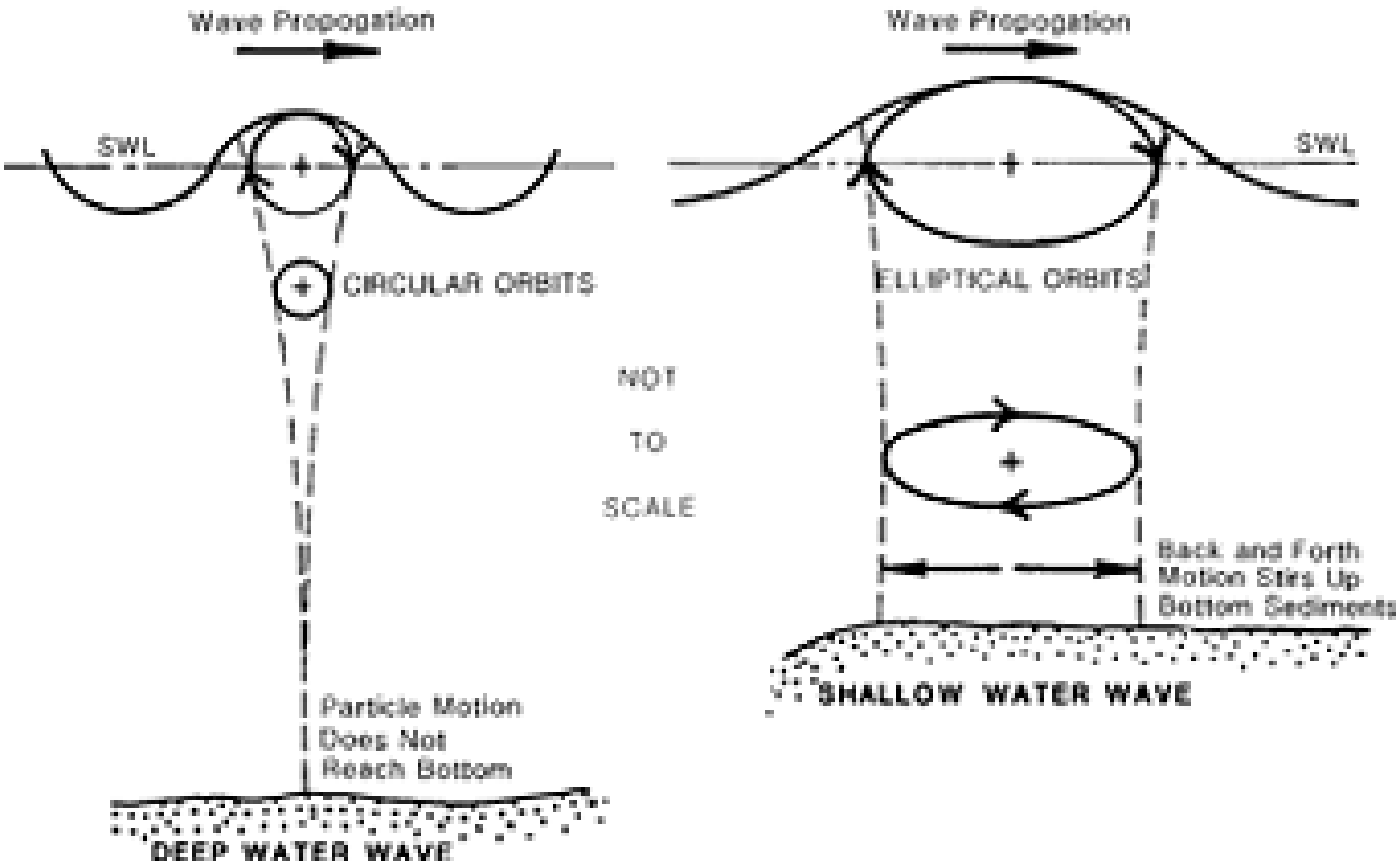


Figure 2: Propagation of a linear wave: water particle trajectories.

2. Linear wave theory: short waves

As previously stated, the wave speed depends on *h*, and such behaviour gives rise to refraction. But, in intermediate and deep waters, the waves speed depends on wave length as well (dispersion):

$$c = \frac{\omega}{k} = \sqrt{\frac{g}{k}\tanh(kh)}$$

Dispersive waves are characterized by the presence of group velocity, defined as the propagation speed of the wave energy:

$$C = \frac{d\omega}{dk} = c - \lambda\frac{dc}{d\lambda}$$

For example, the front of a wave system propagates with the group velocity, while the crests travel with the wave speed; therefore, since for gravity waves the group velocity is lower than the wave speed (in particular, in deep waters *C*=*c*/2), as a result of dispersion, the crests disappear as soon as they reach the wave front.

3. Formulation for long waves and currents

For long waves in shallow waters the wave speed depends on *h* only:

$$c = \frac{\omega}{k} = \sqrt{gh}$$

In this case, the group velocity is equal to the wave speed, and the front of a wave system propagates with the same velocity of the crests. Indeed, the propagation of long waves in shallow waters appear as the rigid motion of a corrugated surface.

The velocity field is nearly constant along the vertical, and the pressure can be assumed hydrostatic. Indeed, in shallow waters the waves can be considered oscillating currents; as a result, the same mathematical formulation can be used both for wave and currents in the coastal region, unless the waves are too short and dispersion features cannot be neglected.

3. Formulation for long waves and currents

$$\frac{\partial \eta}{\partial t} + \nabla \cdot \left[(h+\eta)\vec{u} \right] = 0$$

$$\vec{u} = (u, v)$$

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -g \nabla \eta - \gamma |\vec{u}| \vec{u} + \nu \nabla^2 \vec{u}$$

with γ bottom friction coefficient. This mathematical formulation, with suitable initial and boundary conditions, can be implemented to investigate both waves and currents.

As already stated, particular care is needed for eddy viscosity models. The results shown in the present report have been obtained by the algebraic formulation (*α* calibration constant and u^* friction velocity) :

$$\nu_T = \alpha u^* h$$

The numerical solutions have been validated by the experimental data obtained at APAT laboratory.

3. Formulation for long waves and currents

Figure 3: Wave-harbour interaction in shallow waters.

3. Formulation for long waves and currents

Fig.3 points out the effects of diffraction. This feature occurs when there is a variation of wave height along the wave crest. Diffraction takes place when waves pass an obstacle (of dimension comparable with the wave length) or pass through a gap in a breakwater. Waves tend to turn away from regions of high wave height, and the crests become circular, as they were originated at a point. The importance of wave diffraction results from this direction changes, as it introduces wave energy into regions behind obstacles. This wave property, similar to diffusion, occurs both in deep and shallow waters.

In fig.4 the bathymetry of a sandy coast with protection submerged barrier is shown, while fig.5 describes the flow field related to an incident wave, with the crest parallel to the shoreline, propagating in this geometry. The refraction pattern is clearly pointed out: the waves steepen under the effect of shoaling and the interaction with the rip current, outlined by the velocity and vorticity fields.

3. Formulation for long waves and currents

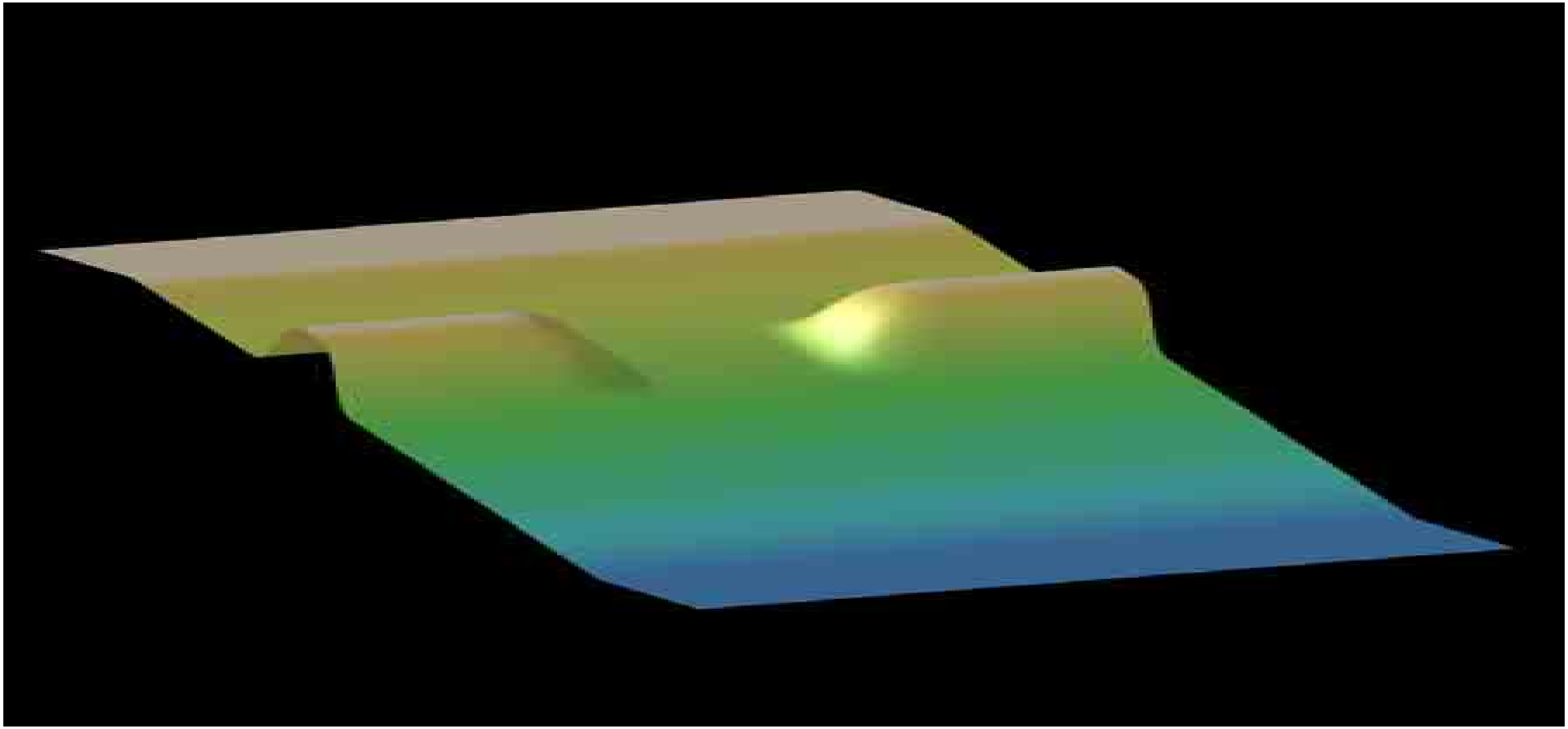
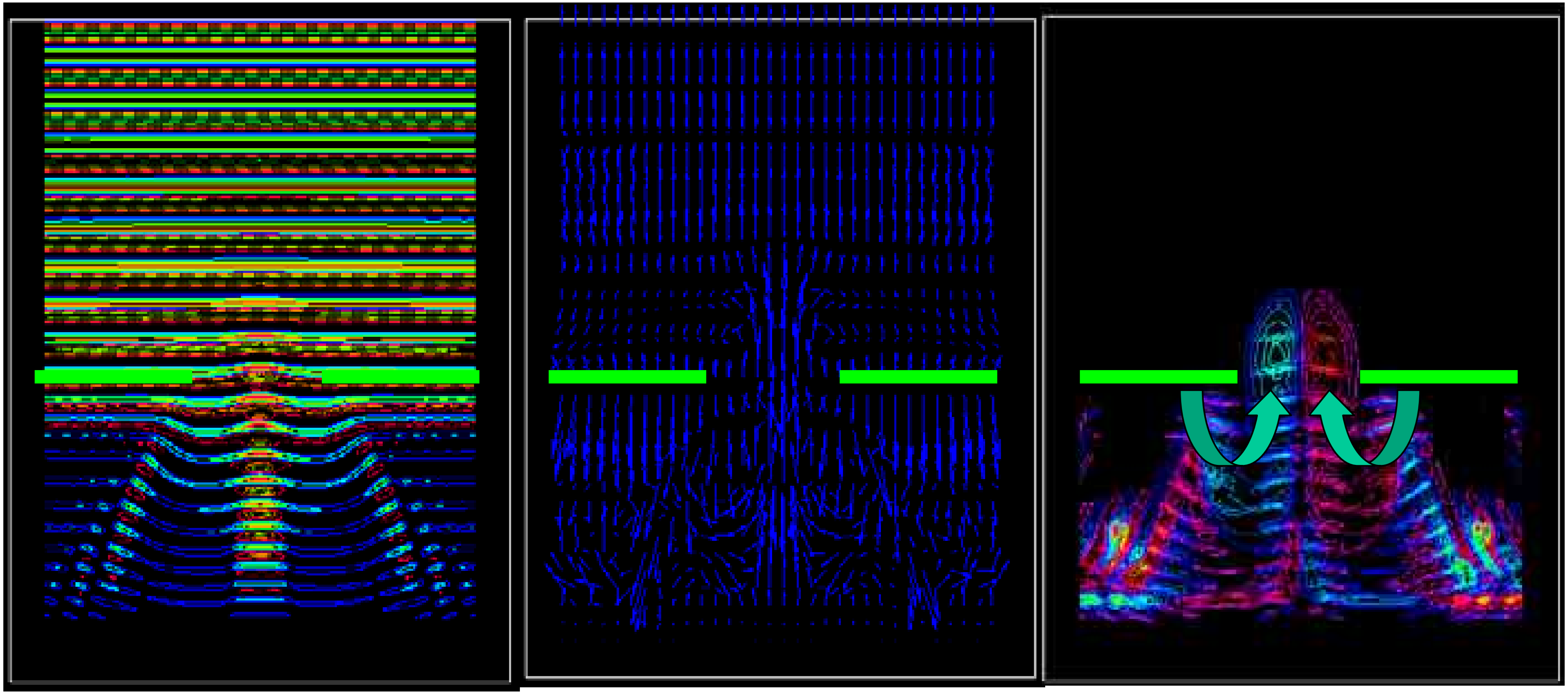


Figure 4: Wave-structure interaction in shallow waters.

3. Formulation for long waves and currents



Free surface Velocity Vorticity

Figure 4: Wave-structure interaction in shallow waters: rip current generation.

4. Jets

4.1 General remarks

A jet is the entry of a fluid in a still environment. As previously pointed out, if the ambient fluid has the same density of the incoming fluid, the jet is of barotropic type, if the densities are different the jet is baroclinic. River mouth and waste water discharge are coastal examples of jets. The hydrodynamic problem of a jet is: how the jet spreads in the ambient fluid? In the 1st chapter it was shown that in the case of barotropic jet the diffusion is related to the generation of vortices, while the baroclinic jet spreads under the effects of buoyancy forces.

In the present chapter the behaviour of a jet in the presence of rigid walls is investigated.

4. Jets

4.2 The Coanda effect

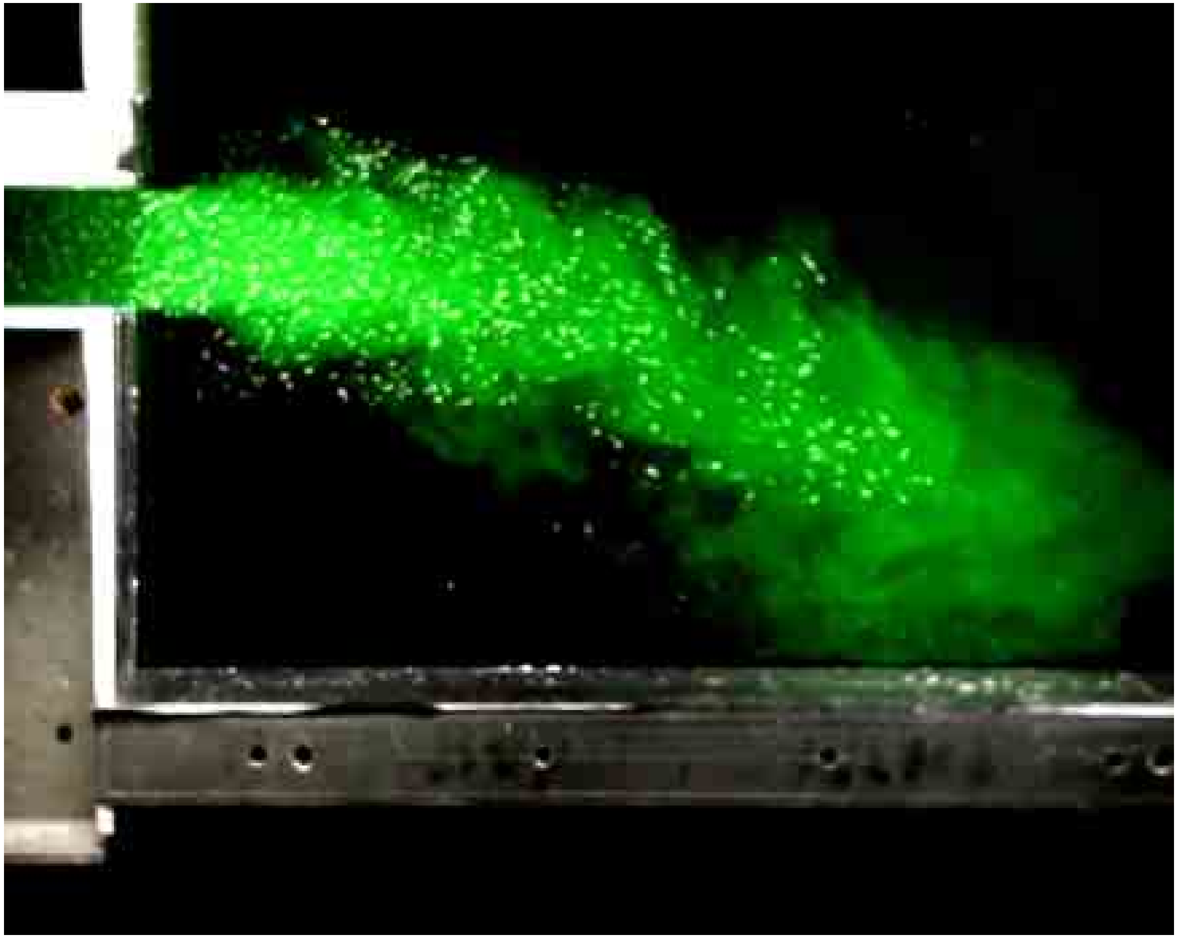


Figure 5: Jet-wall interaction: flow visualization in a laboratory experiment.

4. Jets

Fig.5 shows the interaction of a plane, barotropic jet with a parallel wall. As a result of the interaction, the streamlines are diverted and the jet tends to stick to the wall. This behaviour is called Coanda effect and is due to the generation of a vortex between the jet and the wall: the pressure decrease gives rise to the flow deviation. The flow visualization was performed at APAT laboratory.

Fig.6 shows the velocity fields for some distances of the wall. in this case the numerical results are compared with the experimental data: the agreement is satisfactory.

4. Jets

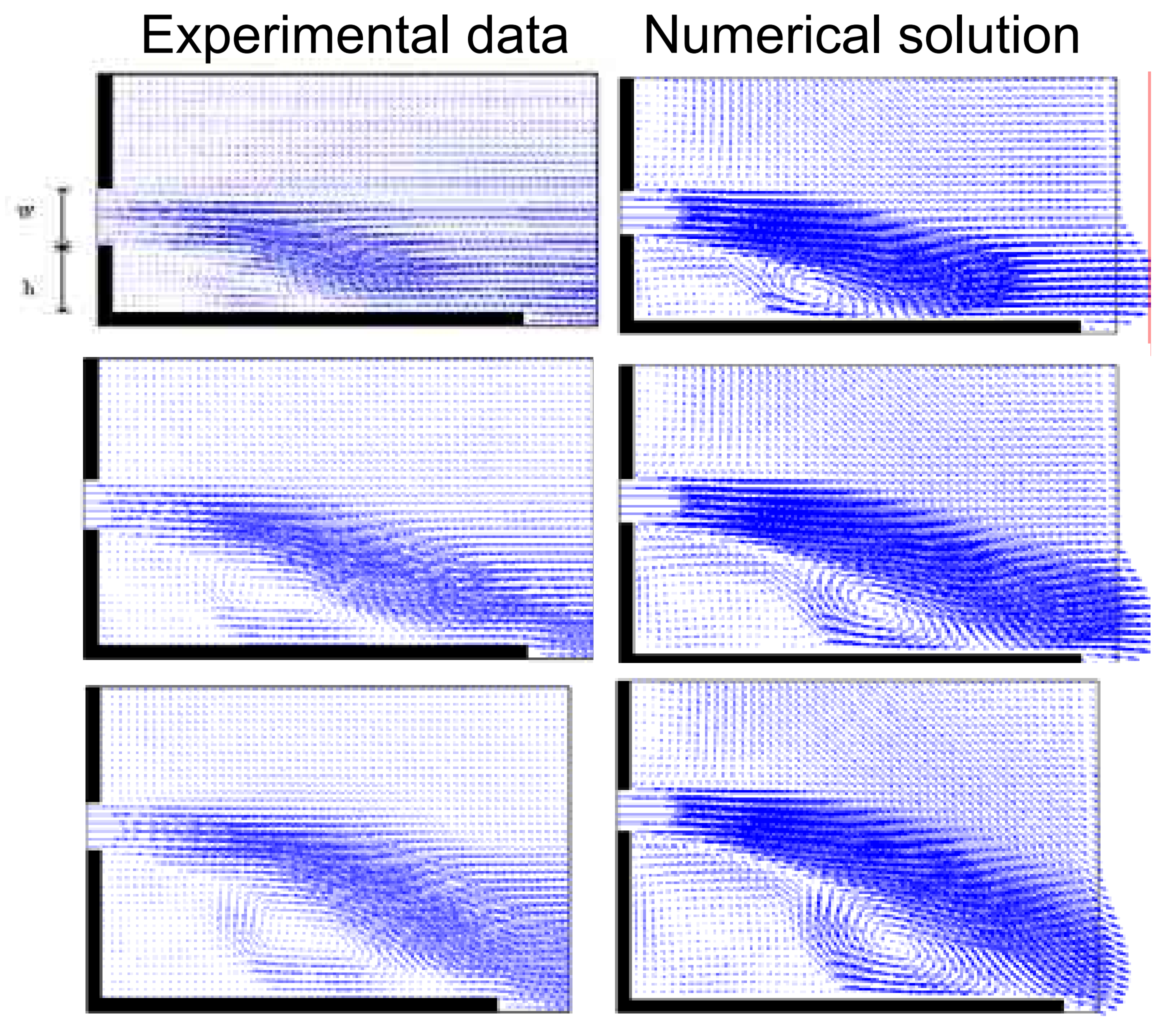


Figure 6: Jet-wall interaction: Numerical computations vs experiments.

4. Jets

4.3 Applications of jet-wall interaction

Numerical solution

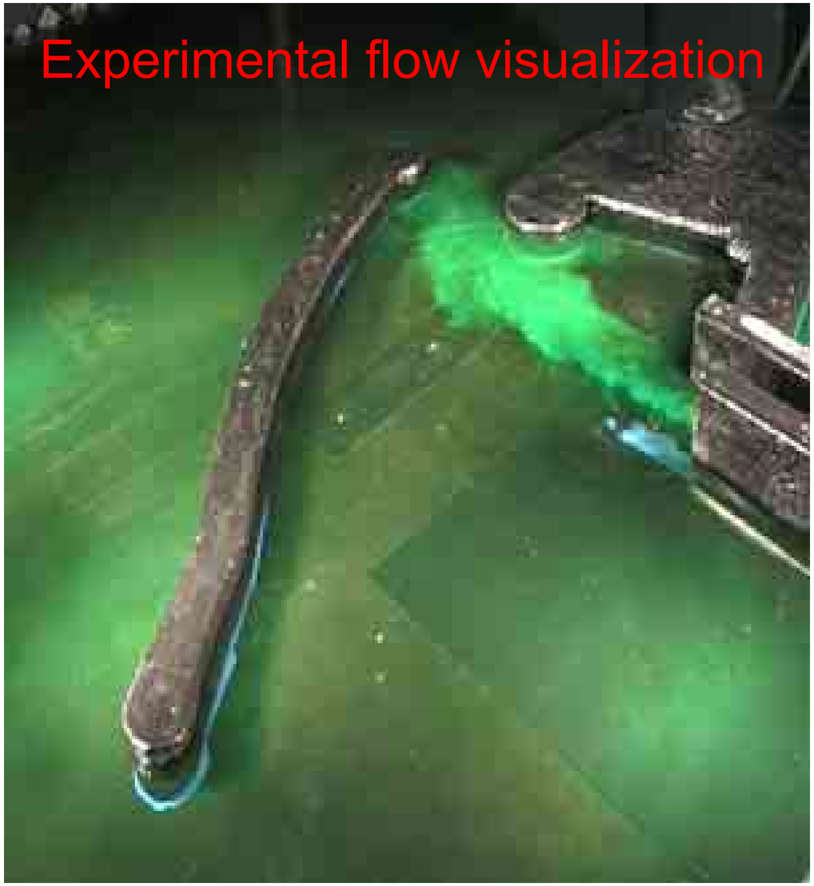


Figure 7a: Jet-wall interaction: Pescara harbour.

4. Jets

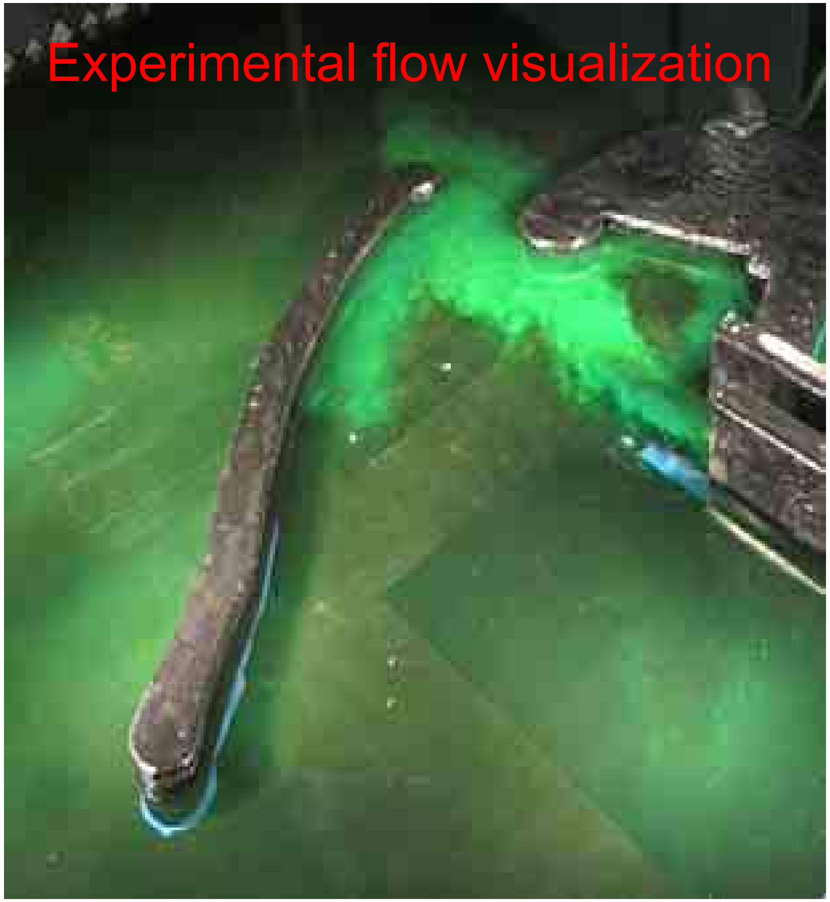


Figure 7b: Jet-wall interaction: Pescara harbour.

4. Jets

Numerical solution

Experimental flow visualization

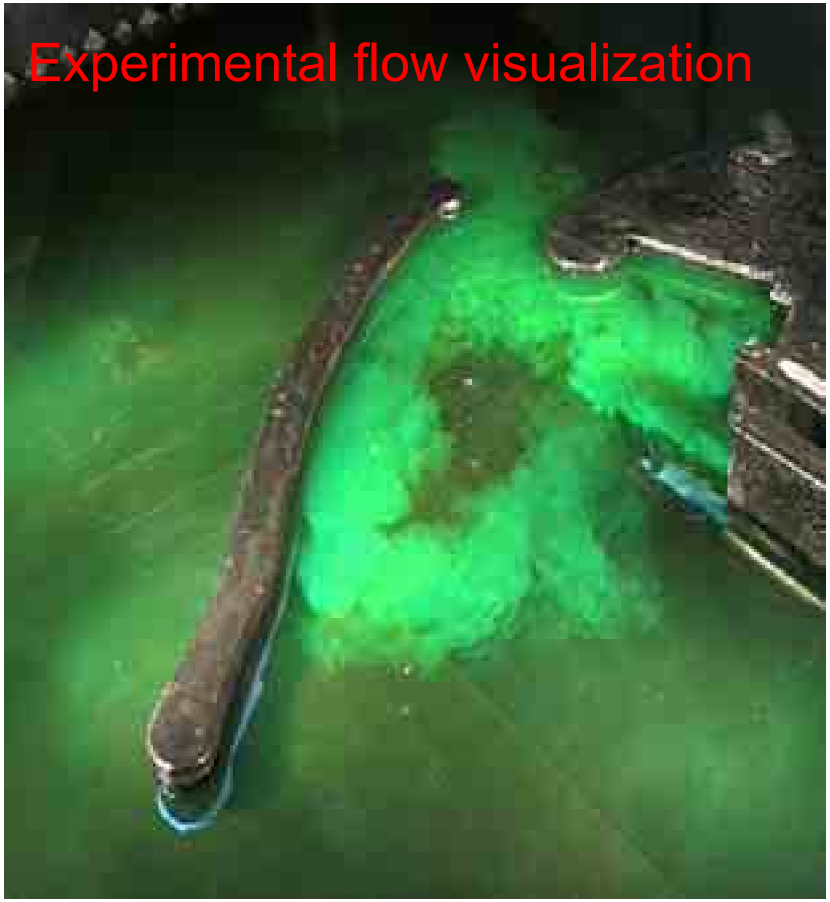
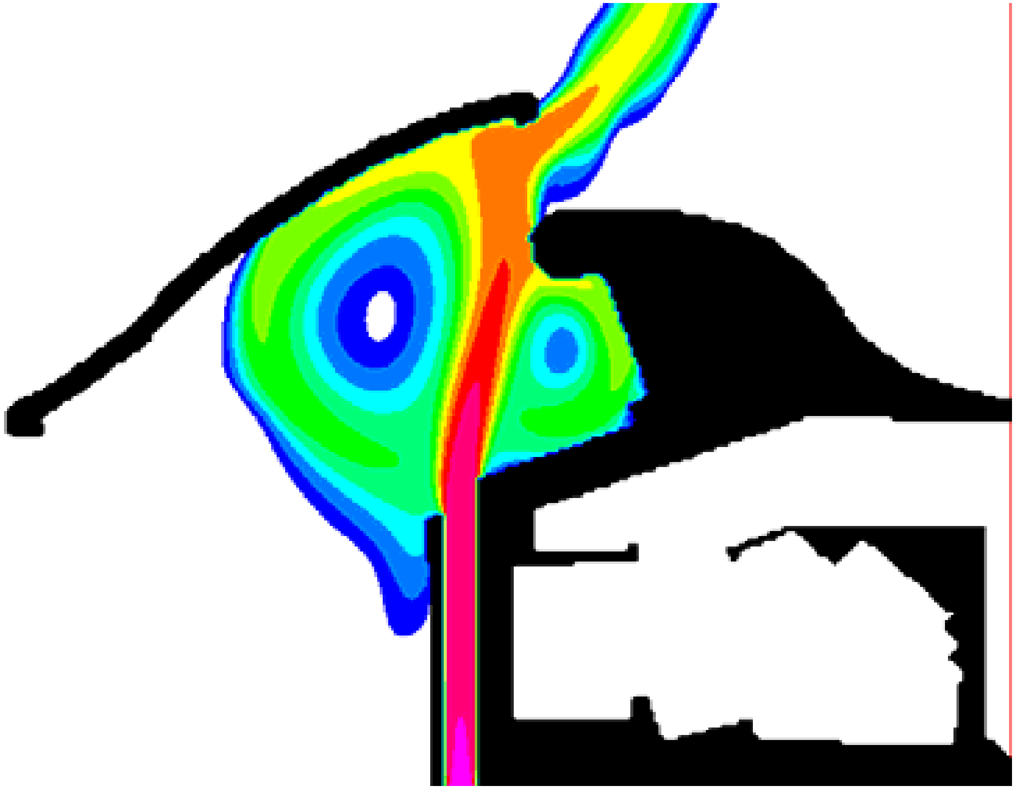


Figure 7c: Jet-wall interaction: Pescara harbour.

4. Jets

Numerical solution



Experimental flow visualization

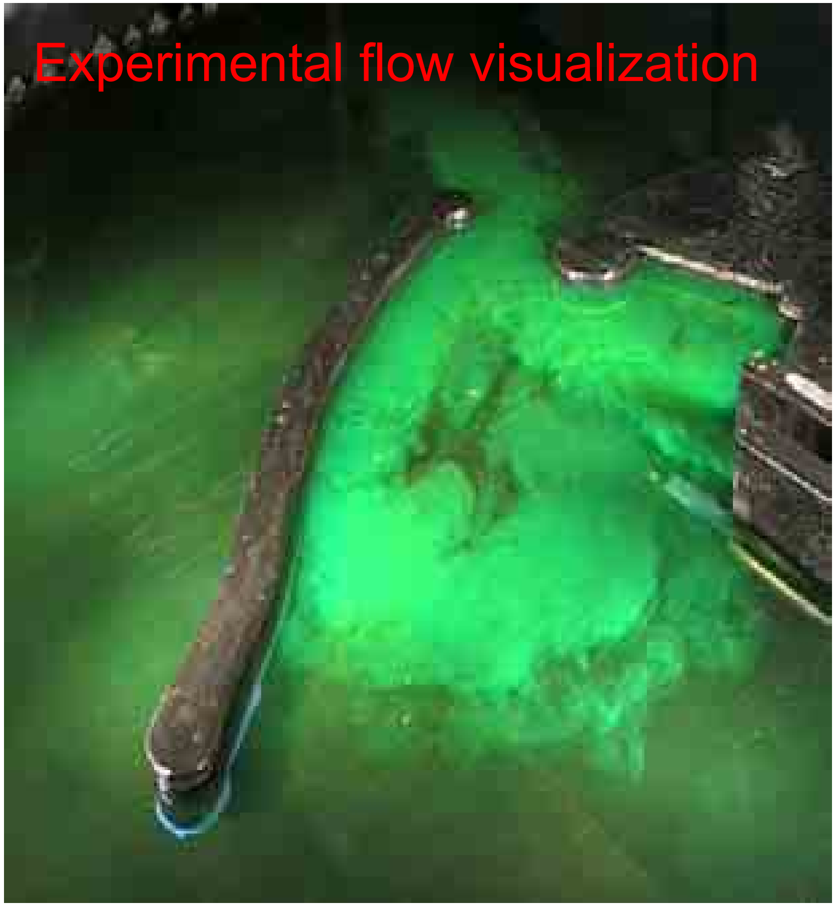


Figure 7d: Jet-wall interaction: Pescara harbour.

4. Jets

Numerical solution | Experimental flow visualization

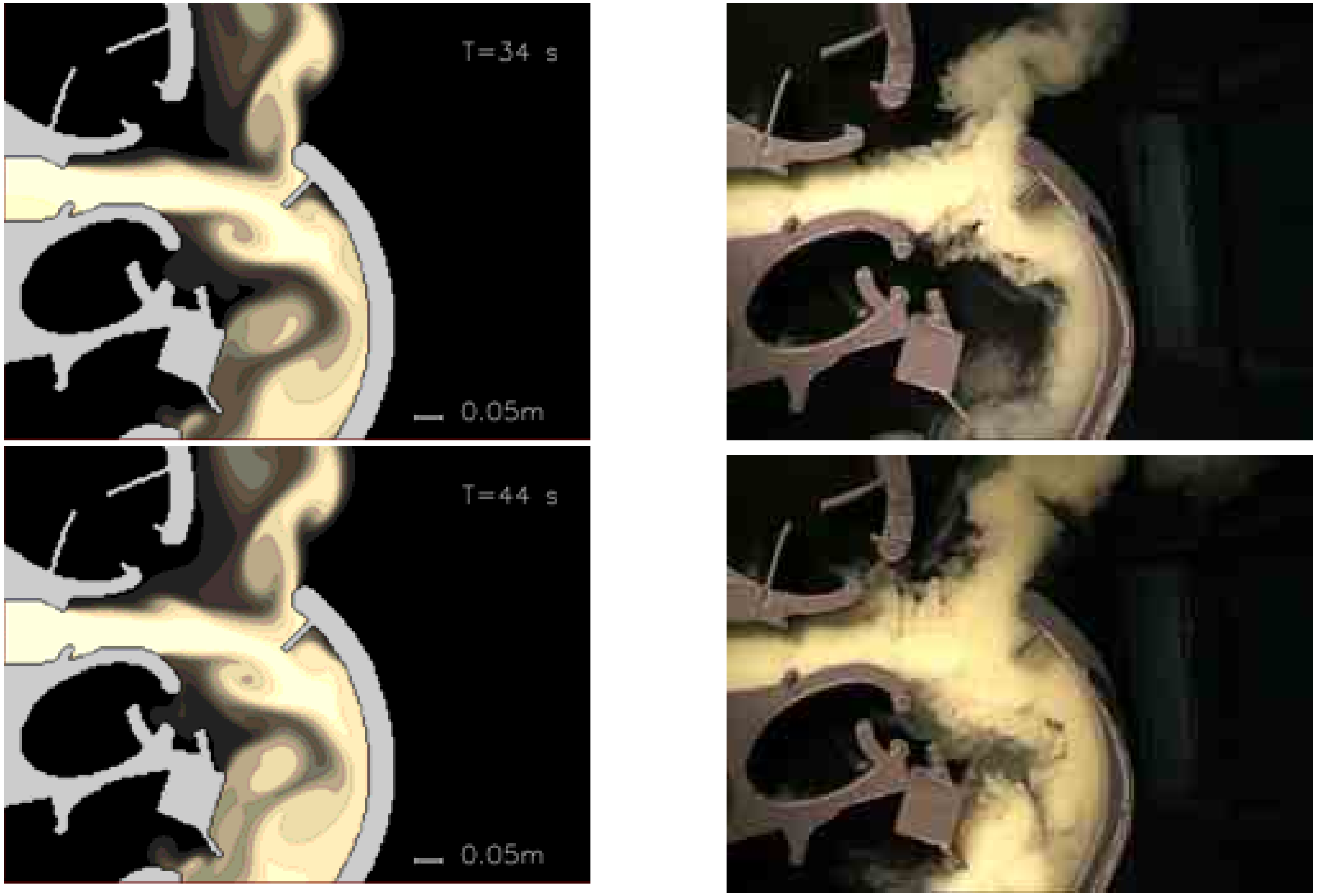


Figure 8: Jet-wall interaction: Latina harbour (Tyrrhenian Sea).